Example

A ball is thrown *upward* from the top of a tower 400 ft high with an initial velocity of 120 ft/sec. Assuming that the acceleration due to gravity is 32 ft/sec/sec *downward* (a fact determined by experiment), determine the velocity with which the ball strikes the ground.

Let us take the upward direction as positive, the time of throwing as $t = 0$, and the origin at the earth's surface. Then

$$a(t) = -32 \text{ ft/sec/sec.}$$

$$v(t) = \int \left(\frac{dv}{dt}\right) dt = \int -32dt = 32t + C.$$

The problem states $v(0) = 120$ ft/sec. Consequently, $120 = 0 + C$, and

$$v(t) = -32t + 120.$$

$$\text{Now } h(t) = \int \left(\frac{dh}{dt}\right) dt = \int v(t)\, dt = \int (-32t + 120)\, dt$$

$$= -16t^2 + 120t + k.$$

The domain of definition of each function is $0 \leq t \leq$ (value of t when the ball strikes the ground).

The problem states $h(0) = 400$. Hence,

$$400 = 0 + 0 + k \quad \text{and} \quad h(t) = -16t^2 + 120t + 400.$$

From here on, the problem is the same as Example 3, Sec. 6-3.

Problem Set 8-1

In all problems involving gravity, assume $g = 32$ ft/sec/sec directed downward.

1. A stone is thrown upward with an initial velocity of 32 ft/sec from the top of a building 560 ft tall. Starting with the assumption that the acceleration of the stone is -32 ft/sec/sec, derive the equation for the height $h(t)$ of the stone from the ground at any time t sec after it is thrown until it strikes the ground. From this, deduce the impact velocity with which the stone strikes the ground.

2. (*a*) Find the velocity of the stone of Prob. 1 as it passes a window 320 ft above the ground level.

(*b*) How high does the stone of Prob. 1 ascend?

3. A pellet is projected upward from ground level with an initial velocity of 96 ft/sec. Using the methods of this section, find an equation giving the distance of the pellet above the earth at any

time t sec after it is projected. When will the pellet reach its highest point? How high will it go? When and with what velocity will it strike the ground?

4. If the pellet of Prob. 3 were projected from a point 80 ft above ground level with an initial velocity of 64 ft/sec, answer the same questions.

5. A ball is dropped from rest from a point 45 ft above the ground. Simultaneously, a second ball is thrown upward from a spot on the ground directly below the first ball. If the initial velocity of the second ball is 30 ft/sec, determine whether or not the two balls will meet while they are still in the air. If they do meet, find the speed, direction, and height of each ball at the moment of impact.

6. A ball is thrown upward with an initial speed of 128 ft/sec. How high above the ground is the ball 3 sec after it is thrown? In which direction is it moving? How high does the ball go? With what velocity does it strike the ground?

7. A bullet is shot upward with an initial velocity of 1,600 ft/sec. How high does it go? How long does it remain in the air? Will it have sufficient velocity when it strikes the ground to be dangerous?

8. David Delbert and Linda Small spent a day at the beach near Big Bugbite Falls. David notes that a piece of wood which was swept over the falls requires 4.5 sec to descend. How high are the falls?

9. David's little brother, who is interested in airplanes, guesses that the piece of wood of Prob. 8 must have been going more than 70 mph as it struck the water near the base of the falls. Is his estimate a reasonable one?

10. A hockey puck travels 216 ft before coming to rest. If the deceleration of the puck is 12 ft/sec/sec, find the initial velocity of the puck.

11. A package slides down a chute 60 ft long with an acceleration of 5 ft/sec/sec.

(*a*) Find the initial velocity of the package if it requires 4 sec to traverse the chute.

(*b*) How fast was the package moving when it was one-third of the way down the chute?

(*c*) How long did it take the package to get halfway down the chute?

(*d*) How far down the chute did the package go during the first half of the time of descent?

12. What uniform acceleration is needed to increase the speed of an automobile from rest (0 mph) to 60 mph in a distance of 440 ft? HINT: First obtain the speeds in feet per second.

**13.* A bullet buries itself 9 in. into a tree in 0.01 sec. Assuming that the deceleration of the bullet was constant and that the bullet came to rest in the indicated time, find the speed of the bullet at the moment of impact.

14. With what approximate speed would a projectile need to be hurled to just reach the top of the Empire State Building, which is 1,250 ft high?

15. A long inclined plane is constructed in such a manner that objects slide down it with an acceleration of 12 ft/sec/sec. An object is thrown *up* the incline. It travels 3,750 feet up the incline before starting to slide back down. Find the initial velocity with which the object was thrown.

16. Work may be defined as force times distance if the force is constant. If the force is not constant but is a function of distance (as in a stretched spring, for example), then we define work as $W = \int_a^b F(x)\,dx$, where $F(x)$ is the force at distance x and W is the work done by $F(x)$ as x varies from a to b. If the force required to stretch a spring x in. from rest is $F(x) = (12x)$ lb, find the work required to stretch the spring from rest. (*a*) 3 in.; (*b*) 6 in.; (*c*) 12 in. In what units will the work be expressed? (*d*) How much work is done in stretching the spring from $x = 7$ to $x = 13$ in.? Set this up as an integral and integrate.

17. Work Prob. 16 if $F(x) = 4x$.

18. A balloon is rising at the rate of 15 ft/sec. A stone dropped from the balloon reached the ground in 8 sec. How high was the balloon? (The nature of the data does not merit an accuracy of more than the nearest ten feet, if that.)

19. A ball is thrown upward and reaches a height of 80 ft in 1 sec. How high will the ball go?

20. A stone is thrown upward with an initial speed of 32 ft/sec from the top of a building 100 ft above ground level. Determine the velocity of the stone as it passes a window 52 ft above ground level on its way down. Start with the assumption that $g = 32$, that is, $s''(t) = -32$ ft/sec/sec and derive all relationships used.

21. Consult a table of integrals (*Handbook of Chemistry and Physics*, for example) and note the variety of things yet to be learned. Find

which integrals we have already studied in this course. Find three functions which are familiar but whose derivative and/or integral we have not studied.

22. Obtain the formula $s = -gt^2/2 + v_0t + s_0$, mentioned earlier in this section. Start with the acceleration due to gravity as a constant g.

23. Imagine you are the navigator on an interplanetary rocket ship about to land on planet J5A. It is known that the rocket-braking power needed for a safe landing is proportional to the gravitational attraction of the planet. Your ship uses $\frac{1}{2}$ rocket-braking power on earth. If on the planet J5A a ball drops 100 ft from rest in 2 sec, can you safely land your ship on J5A?

24. A stone is thrown upward from the top of a building 180 ft high with an initial velocity of 8 ft/sec. Starting with 32 ft/sec/sec downward as the acceleration due to gravity, determine a function which expresses the height of the stone above the ground, as a function of the time t in seconds after the stone is thrown.

25. Determine the velocity of the stone in Prob. 1 as it passes a window 60 ft above ground level.

26. A curve has slope equal to $\frac{7}{2}$ times its abscissa (x value) and passes through $(2,-1)$. Determine the equation of the curve. HINT: If $y' = \frac{7}{2}x$, $y = \int \frac{7}{2}x\,dx = 7x^2/4 + C$. Since $(2,-1)$ lies on the curve $y = 7x^2/4 + C$, it is possible to determine C. Do so.

27. A curve has slope equal to four times its abscissa $[F'(x) = 4x]$ and passes through $(3,-6)$. Determine the equation of the curve.

28. The slope of a curve is six times the square of its abscissa. The curve has a y intercept of 5. Determine the equation of the curve.

29. Verify the theorem $\int x^n\,dx = \dfrac{x^{n+1}}{n+1} + C$ if $n \neq -1$ by differentiation. Be sure to consider the case in which $n = 0$.

30. The slope of a curve at a point is always four units less than twice the abscissa of that point. Find the equation of the family of curves having this property, and pick out the member of the family which passes through $(-2,3)$.

31. If $\dfrac{dy}{dt} = 3t^2 - 7t + 6$ and $y = 30$ when $t = -2$, find an equation expressing y as a function of t. Graph this function.

32. If $\dfrac{dz}{dx} = 8x^7 - 10x^4 + 12x^3 - 7$ and $z = 40$ when $x = 1$, find an equation relating z and x. Make a rough sketch of this equation.