Example

A ball is thrown *upward* from the top of a tower 400 ft high with an initial velocity of 120 ft/sec. Assuming that the acceleration due to gravity is 32 ft/sec/sec *downward* (a fact determined by experiment), determine the velocity with which the ball strikes the ground.

Let us take the upward direction as positive, the time of throwing as $t = 0$, and the origin at the earth's surface. Then

$$a(t) = -32 \text{ ft/sec/sec.}$$
$$v(t) = \int \left(\frac{dv}{dt}\right) dt = \int -32dt = 32t + C.$$

The problem states $v(0) = 120$ ft/sec. Consequently, $120 = 0 + C$, and

$$v(t) = -32t + 120.$$

$$\text{Now } h(t) = \int \left(\frac{dh}{dt}\right) dt = \int v(t)\, dt = \int (-32t + 120)\, dt$$
$$= -16t^2 + 120t + k.$$

The domain of definition of each function is $0 \leqq t \leqq$ (value of t when the ball strikes the ground).

The problem states $h(0) = 400$. Hence,

$$400 = 0 + 0 + k \quad \text{and} \quad h(t) = -16t^2 + 120t + 400.$$

From here on, the problem is the same as Example 3, Sec. 6-3.

Problem Set 8-1

In all problems involving gravity, assume $g = 32$ ft/sec/sec directed downward.

1. A stone is thrown upward with an initial velocity of 32 ft/sec from the top of a building 560 ft tall. Starting with the assumption that the acceleration of the stone is -32 ft/sec/sec, derive the equation for the height $h(t)$ of the stone from the ground at any time t sec after it is thrown until it strikes the ground. From this, deduce the impact velocity with which the stone strikes the ground.

2. (*a*) Find the velocity of the stone of Prob. 1 as it passes a window 320 ft above the ground level.

(*b*) How high does the stone of Prob. 1 ascend?

3. A pellet is projected upward from ground level with an initial velocity of 96 ft/sec. Using the methods of this section, find an equation giving the distance of the pellet above the earth at any

time t sec after it is projected. When will the pellet reach its highest point? How high will it go? When and with what velocity will it strike the ground?

4. If the pellet of Prob. 3 were projected from a point 80 ft above ground level with an initial velocity of 64 ft/sec, answer the same questions.

5. A ball is dropped from rest from a point 45 ft above the ground. Simultaneously, a second ball is thrown upward from a spot on the ground directly below the first ball. If the initial velocity of the second ball is 30 ft/sec, determine whether or not the two balls will meet while they are still in the air. If they do meet, find the speed, direction, and height of each ball at the moment of impact.

6. A ball is thrown upward with an initial speed of 128 ft/sec. How high above the ground is the ball 3 sec after it is thrown? In which direction is it moving? How high does the ball go? With what velocity does it strike the ground?

7. A bullet is shot upward with an initial velocity of 1,600 ft/sec. How high does it go? How long does it remain in the air? Will it have sufficient velocity when it strikes the ground to be dangerous?

8. David Delbert and Linda Small spent a day at the beach near Big Bugbite Falls. David notes that a piece of wood which was swept over the falls requires 4.5 sec to descend. How high are the falls?

9. David's little brother, who is interested in airplanes, guesses that the piece of wood of Prob. 8 must have been going more than 70 mph as it struck the water near the base of the falls. Is his estimate a reasonable one?

10. A hockey puck travels 216 ft before coming to rest. If the deceleration of the puck is 12 ft/sec/sec, find the initial velocity of the puck.

11. A package slides down a chute 60 ft long with an acceleration of 5 ft/sec/sec.

(*a*) Find the initial velocity of the package if it requires 4 sec to traverse the chute.

(*b*) How fast was the package moving when it was one-third of the way down the chute?

(*c*) How long did it take the package to get halfway down the chute?

(*d*) How far down the chute did the package go during the first half of the time of descent?