

Hence the graph of  $f = 300x - 3x^2$  has a horizontal tangent line at  $x = 50$ .

If  $x < 50$ , then  $50 - x > 0$  and  $\frac{dA}{dx} = +$ .

If  $x > 50$ , then  $50 - x < 0$  and  $\frac{dA}{dx} = -$ .

Hence the slope changes from + to 0 to -, indicating a maximum area.

$x = 50$  yd } are the dimensions for the maximum area.  
 $300 - 3x = 150$  yd } The maximum area is 7,500 sq yd.

**Problem Set 4-7**

1. Determine each point at which the curve having as its equation  $y = x^3 - 8x^2 + 30x - 7$  has slope 25.

2. Find an equation of a line which is tangent to the curve given by  $3x^2 + 4y^2 - xy - 18 = 0$  at the point  $(2, -1)$ .

3. A closed rectangular box with base twice as long as it is wide is to be built to contain 72 cu ft. Determine its dimensions if the material used in the box is to be a minimum and no allowance is made for the thickness of material along the edges.

4. Find  $k$  so that  $y = 4x^2 + kx + 9$  will be tangent to the  $x$  axis.

5. A tin cylindrical can is to be constructed to contain 125 cu in. Determine the dimensions of a can requiring the least total sheet metal to construct if the circular top and bottom are cut from square pieces of tin with the scraps wasted and the side is cut from large sheets without waste. (Theorem 3, Sec. 4-4, does not apply.)

6. The yearbook committee in a certain school feels that about 600 people will buy yearbooks if the price is \$5 each. Their experience suggests that for each 5 cents the price is raised, four of the 600 people will decide not to purchase a yearbook. (a) What price will bring in the greatest total income? (b) If the yearbooks cost \$3.50 to produce, what price will bring in the greatest profit? (c) Compare the answers of (a) and (b) and make graphs to explain these facts to the editorial staff, who may understand little mathematics.

7. A telephone answering service for doctors charges each doctor using the service a fee of \$3 per month. They have 500 subscribers. The owners believe that for each cent they raise the cost, one doctor will discontinue the service; that is, if the fee is raised to \$3.25 per month, 25 of the 500 doctors would stop using the service;

if they raised the fee to \$3.30 per month, 30 doctors would drop, etc. On the basis of this assumption, what service fee will bring in the greatest monthly income?

8. A boat company agrees to transport a group of students on an outing for a charge of \$6 per student if 75 or fewer students go on the outing. The boat company further agrees that for each student more than the basic 75, they will reduce the charge 5 cents per student for all students. What number of students will bring in the largest income to the boat company? How much will each student pay in this case?

9. Jack Trot has a 120-apartment building. His experience leads him to believe that he can rent all 120 apartments at \$40 per month for each apartment. He estimates that for each dollar he raises the rent, there will be two empty apartments. That is, if he charges \$41, he will only be able to rent 118 of the apartments; if he charges \$42 per month, he will only rent 116 apartments, etc. What rental will bring him the greatest income?

10. Given  $y = x^3 - 3x^2 - 9x + 5$ . Find maximum and minimum points. Make a table and sketch the locus.

11. Calvin Butterball wishes to enclose a rectangular area and divide it into three pens. Calvin decides to use an existing fence for one side of the rectangle and make the cross fences perpendicular to the existing fence as shown in Fig. 4-19. Find the largest total area Cal may enclose with 200 ft of additional fence.

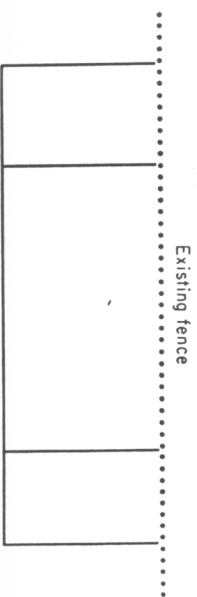


Figure 4-19

12. A box with a square base and open top is to be constructed to hold 4 cu ft. Determine the dimensions of such a box having the least surface area.

13. Silas Hogwinder wishes to enclose a rectangular plot of ground and divide it into four pens by erecting three fences parallel to one side of the rectangle. If Silas has a total of 300 ft of fencing, what is the largest area he may enclose?