

We next turn our attention to further applications. No new concepts or techniques are involved.

### Problem Set 4-11

In these problems, you may wish to use the following facts: (1) If  $y = 1/x$ ,  $y' = -1/x^2$ . (2) If  $f(x) = 1/x^2$ ,  $f'(x) = -2/x^3$ . These facts may be obtained by use of the delta process; however, you may assume them here. They were derived in Probs. 4 and 5 of Set 4-2.

- The sum of two numbers is 12. Find the numbers if their product is to be as large as possible. **HINT:** Let  $x$  and  $(12 - x)$  be the numbers. The problem is then to determine  $x$  and  $(12 - x)$  such that  $P = x(12 - x)$  is as large as possible and to prove conclusively that the values given do make the product as large as possible.
- Determine a positive number such that the sum of the number and its reciprocal is as small as possible.
- A closed rectangular box with base twice as long as it is wide is to be built to contain 72 cu ft. Determine its dimensions if the material used in the box is to be at a minimum and no allowance is made for the thickness of the material.

4. (a) Hobart Musson wishes to enclose a rectangular plot of ground and divide it into four pens by erecting three fences parallel to one side of the rectangle. If Hobart has a total of 600 ft of fencing, what is the largest area he can enclose?

(b) Do the four pens need to be of the same size?

5. (a) The Bilgewater Beach is an apartment hotel, containing 125 apartments. The owner's experience leads him to believe that he can rent all 125 apartments at \$60 a month for each apartment. He estimates that for each \$1 he raises the rent there will be two empty apartments; that is, if he charges \$61 a month he will be able to rent only 123 of the apartments. If he charges \$62 a month he will rent only 121 apartments, and so on. What rental will bring him the greatest total income?

(b) Rework if there are 110 apartments in all.

(c) Rework if there are only 30 apartments in all.

6. The Norman Bus Company agrees to transport a group of students on an outing for a charge of \$6 per student if 75 or fewer

students go on the outing. The bus company further agrees that for each student more than the basic 75 they will reduce the charge 5 cents per student for all the students who are going on the trip. What number of students will bring the largest income to the bus company? How much will each student pay in this case?

7. In the proof of a problem a student uses the statement, "Since a real number plus its square is never negative, it follows that . . . ." Show that the student's proof is not valid.

8. If  $x$  is a real number, what is the smallest value of the function  $f(x) = x^2 + 1/x^2$ ?

9. What is the area of the largest isosceles triangle having its equal sides of length 100 cm?

10. (a) A cylindrical container with open top is to have a volume of  $125\pi$  cu in. Find the dimensions of the container using the smallest total amount of metal.

(b) Generalize the problem of part (a) to a container of volume  $V$ . Determine the ratio of diameter to height.

11. (a) Find the greatest and least values of the fraction having its numerator 25 greater than the square of its denominator.

(b) Same as (a), but restrict the fraction to be positive.

12. Find the  $x$  coordinates of all points at which the given curves  $y = x^3 - 2x^2 - 5x + 17$ , and  $y = \frac{2}{3}x^3 - 3x^2 + 10x - 11$  have parallel tangents corresponding to the same values of  $x$ .

13. A block of ice slides down a chute 100 ft long with an acceleration of 16 ft/sec/sec. The distance of the block from the bottom of the chute is given by  $s(t) = -8t^2 - 20t + 100$ .

(a) At what time does the block reach the bottom of the chute?

(b) How long does it take the block to slide 4 ft; that is, when does  $s(t) = 96$ ?

14. A ball rolls on an inclined plane so that its distance  $s$  ft from the bottom of the plane at time  $t$  sec is  $s(t) = 6t - 2t^2$ .

(a) How far up the inclined plane will the ball roll?

(b) At what time does the ball reach the bottom of the inclined plane?

(c) At what times is the ball 1 ft from the bottom of the plane?

15. The point  $(0, -4)$  does not lie on the curve  $y = x^2$ . Nevertheless, there are two lines passing through  $(0, -4)$  which are tangent to the curve  $y = x^2$ . Find the equations of these two lines (Fig.