Substituting the known values, we obtain

$$500 = 96(5)\frac{dh}{dt}$$

and $\dfrac{dh}{dt} = \dfrac{500}{96(5)} = 1.04$ in./min.

Problem Set 6-4

1. Gas is being forced into a spherical balloon at the rate of 400 cu in./min. How fast is the radius of the balloon increasing when the radius is 5 in.?

2. Same as Prob. 1 for a 10-in. radius.

3. Gas is forced into a spherical balloon at the rate of 600 cu in./min. How fast is the radius of the balloon increasing when it encloses 288π cu in. of gas?

4. At what rate is the surface area of the balloon of Prob. 1 changing when $r = 5$ in.?

5. At what rate is the surface area of the balloon of Prob. 3 changing at the instant in question?

6. Determine the rate at which the depth of the water is changing in the trough of Example 2, Sec. 6-4, when the water is 1 in. deep.

7. Same as Prob. 6 when the water is 10 in. deep.

Discuss and sketch the functions given in Probs. 8 to 15. List all maximum, minimum, and inflection points and the approximate intercepts.

8. $y = 2x^3 - 7x + 5.$

9. $y = 3x^2 - 4x + 2/x.$

10. $y = 2x^2 + 11.$

11. $y = x^3 - 12x.$

12. $y = 4x^3 - 108.$

13. $y = 12 - x^3.$

14. $y = 5 + 3x - 10x^3.$

15. $y = -7 - 2x + x^2.$

16. A steamer is rented for an excursion for 100 passengers at $10 per passenger providing 100 passengers purchase tickets. If fewer than 100 tickets are sold, the rental remains at $1,000. The owners agree to reduce all fares 5 cents per fare for each person beyond the basic 100. (*a*) How much is the maximum rental the owners may expect on the excursion? (*b*) How many passengers will need to go to obtain this maximal rental? (*c*) Assuming that the fire laws will not permit the steamer to carry more than 180 passengers, what will be the smallest gross income the owners may receive for a legal load? (*d*) Sketch a graph of income I vs. number of passengers x. Note that, for $0 < x \leqq 100$, I is constant.

17. An exporter can ship a cargo of 100 tons today at a profit of $5 a ton. By waiting, he can add 20 tons per week to the shipment, but the profit on all that he exports will be reduced 25 cents per ton per week. How long will it be profitable to wait?

18. A light is placed at ground level 50 ft from a high wall. A man 5 ft tall starts at the light and walks directly toward the wall at the rate of 8 ft/sec. How fast is the top of his shadow moving down the wall when he is halfway there?

19. The demand for an article varies inversely as the $\frac{5}{2}$ power of the selling price. If the articles cost $1 each to manufacture, determine the selling price which will produce the greatest profit.

20. The amount of candy and pop an individual huckster can sell at a football game decreases as the number of hucksters increases. If 100 hucksters can average $80 each in sales at a big game, and if for each additional huckster added the average per huckster drops 10 cents, find the number of hucksters for a maximum total income for the concession owner. How much does each huckster sell at this maximum? How much larger is the maximum total income than if 100 hucksters were employed?

21. John Garnett is in the hospital recovering from a bike accident. A doctor is inflating a spherical balloon to please John. If hydrogen is supplied at 100 cu in./min, how long will it take to inflate the balloon to a radius of 10 in.? How fast is the radius increasing when the balloon is 5 in. in radius? 10 in. in radius? Just before it bursts at 12.3 in. in radius?

22. Show that $y = x^3 - 6x^2 - 15x + 8$ is neither rising nor falling at the points where $x = -1$ and $x = 5$.

23. Consider the curve of Prob. 22. Tell whether the curve is rising or falling as x increases on each of the intervals (*a*) $x < -1$, (*b*) $-1 < x < 5$, (*c*) $5 < x$.

24. Determine the critical points and the intervals in which $y = (x + 6)(x - 1)^3$ is increasing as x increases.

25. A tin container is to be constructed in the form of a right circular cylinder containing 27 cu in. volume. If the top and bottom of the can are cut from square sheets and the corner pieces are wasted, find the dimensions of the container requiring the least tin.

26. Bill McFurson wishes to fence in a rectangle of ground in his pasture and then divide the rectangle into three (not necessarily equal) pens, by erecting two fences parallel to the ends of the