

## EXERCISES

In Exercises 1 through 14, use the shell method to find the volume of the solid generated by revolving the region with the given boundary about the indicated axis.

1. Bounded by  $y = x^2$ ,  $y = 1$  about the line  $y = 1$
2. Bounded by  $y = x^2$ ,  $y = 1$  about the line  $x = 2$
3. Bounded by  $y = x$ ,  $y = x^2$  about the  $y$ -axis
4. Bounded by  $y = x$ ,  $y = x^2$  about the  $x$ -axis
5. Bounded by  $y = \sqrt{x}$ ,  $x = 4$ ,  $y = 0$  about the  $y$ -axis
6. Bounded by  $y = \sqrt{x}$ ,  $x = 4$ ,  $y = 0$  about the line  $x = 5$
7. Bounded by  $y = \sqrt{x}$ ,  $y = x^2$  about the  $y$ -axis
8. Bounded by  $y = \sqrt{x}$ ,  $y = x^2$  about the  $x$ -axis
9. Bounded by  $x = y^2$ ,  $y = x - 2$  about the line  $y = 2$
10. Bounded by  $y = x^2$ ,  $y = 3 - 2x$  about the line  $x = 1$
11. Bounded by  $y = \cos x$  for  $-\pi/2 \leq x \leq 3\pi/2$ ,  $y = 0$  about the  $y$ -axis
12. Bounded by  $y = \sin x$  for  $0 \leq x \leq \pi$ ,  $y = 0$  about the line  $x = -1$
13. Bounded by  $y = \sin x$  for  $0 \leq x \leq \pi$ ,  $y = 0$  about the line  $x = 2\pi$
14. Bounded by  $y = \cos x$  for  $-(\pi/2) \leq x \leq \pi/2$ ,  $y = 0$  about the line  $x = 2$
15. Use the method of cylindrical shells to derive the formula  $V = \frac{1}{3}\pi r^2 h$  for the volume of a right circular cone of height

$h$  with base of radius  $r$ . [Hint: Revolve the region bounded by the lines  $y = (r/h)x$ ,  $y = 0$ , and  $x = h$  about the  $x$ -axis.]

16. Derive the formula  $V = \frac{4}{3}\pi a^3$  for the volume of a sphere of radius  $a$ , using the method of cylindrical shells.
  17. Use the method of cylindrical shells to find the volume of the doughnut (torus) generated by revolving the disk  $x^2 + y^2 \leq a^2$  about the line  $x = b$  for  $b > a$ . [Hint: Part of the integral can be evaluated as the area of a familiar figure.]
- In Exercises 18 through 24, use the method of cylindrical shells with a calculator or computer and numerical techniques (Simpson's rule with  $n \geq 10$ , Newton's method). Find the volume of the solid generated by revolving the region with the given boundary about the indicated axis.
18. Bounded by  $y = \tan x$ ,  $y = 0$ ,  $x = \pi/4$  about the  $y$ -axis
  19. Bounded by  $y = \sec x - 1$  for  $-(\pi/2) < x < \pi/2$ ,  $y = 1$  about the line  $x = 2$
  20. Bounded by  $y = \sin \sqrt{x}$  for  $0 \leq x \leq \pi^2$ ,  $y = 0$  about the  $y$ -axis
  21. Bounded by  $y = 3^x$ ,  $x = 0$ ,  $x = 2$ ,  $y = 0$  about the line  $x = 3$
  22. Bounded by  $x = 2^y$ ,  $y = 0$ ,  $y = 3$ ,  $x = 0$  about the line  $y = -1$
  23. Bounded by  $y = 10/(x^2 + 1) - 2$ ,  $y = 0$  about the line  $x = 3$
  24. Bounded by  $y = x^2$ ,  $y = \sin x$  about the  $y$ -axis

\* Used INTEG by Pa B!