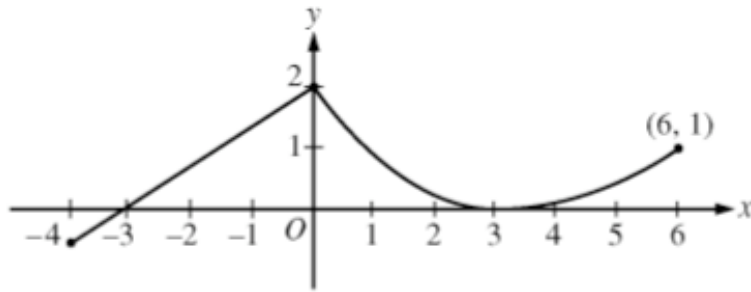

You know the directions . . .

| | | | | | |
|--------|---|---|----|---|----|
| x | 2 | 3 | 5 | 8 | 13 |
| $f(x)$ | 1 | 4 | -2 | 3 | 6 |

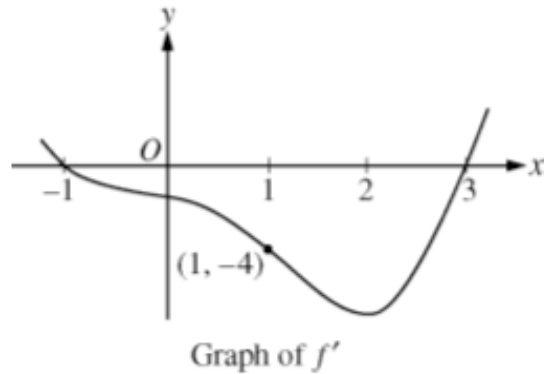
1. Let f be a function that is twice differentiable for all real numbers. The table above gives selected points in the closed interval $[2,13]$

- (a) Estimate $f'(4)$. [Show the work that leads to your answer.]
- (b) Find the average rate of change for the interval $[2,13]$. [Show all work]
- (c) Show that there must be a $c, 2 < c < 13$ such that $f'(c) = \frac{5}{11}$ [Show all work, justify, etc.]
- (d) Use a left Riemann sum with subintervals indicated by the data in the table to approximate $\int_2^{13} f(x) dx$ [Show all work]



Graph of f

2. A continuous function f is defined on the closed interval $[-4, 6]$. The graph of f consists of a line segment and a curve that is tangent to the x -axis at $x = 3$ as shown above. On the open interval $(0, 6)$ the function f is twice-differentiable with $f'' > 0$.
- (a) Is f differentiable at $x = 0$? [Use the definition of the derivative with one-sided limits to justify your answer.]
- (b) For how many values of a , $-4 \leq x \leq 6$, is the average rate of change of f on the interval $[a, 6]$ equal to 0? Give a Calculus-based reason for your answer.
- (c) Find $\int_{-3}^0 f(x) dx$. [Remember to show all work]



3. Let f be a twice-differentiable function defined on the open interval $(-1.2, 3.2)$. The graph of the derivative of f is shown above. The graph of f' crosses the x -axis at $x = -1$ and $x = 3$ and has a horizontal tangent at $x = 2$.
- (a) If $f(1) = 4$, then write the equation of the tangent line to the graph of f and use the tangent line equation to estimate a value of $f(0.9)$
- (b) For $-1.2 < x < 3.2$, find all values of x for which f has a relative extrema. [Justify your answer. Yes, that means with sentences!]
- (c) For what value(s) of x does the graph of f have a point of inflection? [Justify]
- (d) Is the tangent line to the graph of f at $x = 1$ lie above or below the curve [of f]? [Justify]

4. Consider the closed curve in the xy -plane given by $x^2 + 2x + y^4 + 4y = 5$.

(a) Show that $\frac{dy}{dx} = \frac{-(x+1)}{2(y^3+1)}$

(b) Find the coordinates of the two points on the curve when the line tangent to the curve is vertical. [Show all work]

5. Many people use UPS for sending their packages because it will accept a box for shipment if the sum of its length and girth does not exceed 165 inches. What dimensions will give a box with a circular end the largest possible volume? Show all work and feel free to use your TI.