The negative velocity indicates the distance from the ground $h(t)$ is decreasing; that is, the stone is going downward.

Problem Set 6-3

In Probs. 1 to 5, determine the position, velocity, and acceleration (at the indicated time) of a particle which moves along the s axis and whose directed distance from the origin is given by $s(t)$. Describe the motion of the particle for $-100 \leq t \leq 100$.

1. $s(t) = t^2 - 3t + 5$ at $t = 1$, $t = 7$, and $t = 11$.
2. $s(t) = t^4 + 4t - 9$ at $t = 1$, $t = 0$, and $t = -3$.
3. $s(t) = 64t - 16t^2$ at $t = 0$, $t = 5$, and $t = 10$.
4. $s(t) = 17t$ at $t = 0$, $t = 5$, and $t = 7$.
5. $s(t) = 11$ at $t = 0$, $t = 5$, and $t = 15$.

In Probs. 6 to 10, determine the times at which the particle has the given velocity or acceleration and determine the distances of the particle from the origin at these times.

6. $s(t) = 10t^3 + 7t^2 + 3$ where $a(t) = 2$.
7. $s(t) = 12t^3 - 3t + 5$ where $v(t) = 6$.
8. $s(t) = t^4$ where $v(t) = 32$.
9. $s(t) = 3t^3 - 7t + 2$ where $a(t) = 36$.
10. $s(t) = 17 + t - t^2$ where $v(t) = 9$.

11. A hockey puck is struck at time $t = 0$. The distance in feet of the puck from the spot at which it was struck is $s(t) = 72t - 3t^2$, where t is in seconds. How far did the puck travel before coming to rest? In what t interval does the function have meaning in this problem?

12. A stone is thrown upward from the top of a tower 100 ft above the ground with an initial velocity of 64 ft/sec. The height h of the stone above the ground t sec after it is thrown is given by $h(t) = -16t^2 + 64t + 100$. Determine the velocity of the stone as it passes a window 20 ft above ground level.

13. From a point 1,200 ft above the earth's surface, a stone is thrown upward with a speed of 160 ft/sec. Find the impact velocity if the height of the stone at any time t after the stone is thrown is

$$h(t) = -16t^2 + 160t + 1{,}200.$$

14. A block of ice slides down a chute 100 ft long with an acceleration of 16 ft/sec/sec. The distance of the block from the bottom of the chute is given by

$$s(t) = -8t^2 - 20t + 100.$$

Find the velocity of the block at the bottom of the chute; that is, when $s(t) = 0$.

15. How fast is the block of Prob. 14 sliding when it has slid 4 ft?

16. A ball rolls on an inclined plane so that its distance s ft from the bottom of the plane at time t sec is $s(t) = 6t - 2t^2$. How far up the inclined plane will the ball roll?

17. At what time and with what speed does the ball of Prob. 16 reach the bottom of the inclined plane?

18. What is the speed of the ball of Prob. 16 when it is 1 ft from the bottom of the plane?

19. John Garnett is coasting down a long hill on his bike. His distance (in feet) from the top of the hill at any time t in seconds is given by $s(t) = 5t^2 + 7t$. After John has coasted 160 ft from the top of the hill, he strikes a large stone and falls from his bike. Determine his velocity at the moment he strikes the stone. Express this velocity in miles per hour. Do you think it likely that John was hurt in the fall?

20. If John had fallen when he had coasted only half as far as in Prob. 19, with what velocity would he have fallen?

6-4. General Rate of Change

The remarks of Sec. 6-3 on rate of change are not limited to time rate of change of distance. Other rates of change are handled in a similar manner. The change in volume with respect to time, for example, is $dV/dt = \lim_{\Delta t \to 0} \Delta V/\Delta t$.

Example 1

Hydrogen is being forced into a spherical balloon at the constant rate of 1,000 cu in./min. How fast is the radius of the balloon increasing when the balloon is 2 ft in radius?

The volume V of a sphere is $V = \frac{4}{3}\pi r^3$. We are given $\frac{dV}{dt} = 1{,}000$ cu. in./min and are asked to find $\frac{dr}{dt}$ when $r = 2$ ft $= 24$ in. Then, since $r = r(t)$, using Sec. 4-10, we have

$$\frac{dV}{dt} = \frac{dV}{dr}\cdot\frac{dr}{dt} = \frac{d\frac{4}{3}\pi r^3}{dr}\cdot\frac{dr}{dt} = 4\pi r^2\frac{dr}{dt}.$$